voltage, kV; G, weight of spheroid; α_0, heat-transfer coefficient in the absence of field; φ, ratio of times for complete evaporation without and with fields (or inverse ratio of corresponding heat-transfer coefficients); ε, liquid dielectric constant; σ, surface tension of liquid.

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REYNOLDS ANALOGY FOR BOILING

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A Reynolds analogy for boiling is proposed and a generalized heat-transfer equation for bubble boiling in tubes and in a large volume is presented.

Contemporary knowledge of heat transfer and hydrodynamics for bubble boiling of liquids is based mainly on theoretical and experimental investigations of the boiling process in a large volume. The general correlations developed by Soviet investigators (Kruzhilin, Kutateladze, Tolubinskii, Borishanskii, et al.), can be used to design complex vaporizing equipment for boiling a liquid in a large volume. In most contemporary heat exchangers, boiling does not occur in a large volume, but in tubes and channels. However, there are as yet no well-founded reliable general formulas to calculate the boiling process in tubes with natural and forced flow circulation. The available correlations for boiling in a large volume are not suitable for calculating boiling in tubes, since they do not contain the necessary parameters and conditions to account for the characteristic general and special processes. The analogy of the Reynolds number Re* does not take into account all the special hydrodynamic features of the process of boiling in tubes.

We recall that the parameter

$$\mathrm{Re} = \frac{wd_0}{\nu} \tag{1}$$

was proposed by Reynolds to describe the motion of a single-phase medium in tubes; here the velocity w is the ratio of the mass flow to the area of the flow or the tube, f: $w = V/f$ ($\mathrm{m^3/h/m^2 = m/h}$).

To describe motion of a two-phase fluid with boiling, Kichigin has proposed [1] a Reynolds analogy

$$\mathrm{Re}^* = \frac{q/L\rho_V}{\nu}\left[\frac{\sigma}{g(\rho-\rho_V)}\right]^{0.5}. \tag{2}$$

In Eq. (2), which differs appreciably from Eq. (1), the velocity appears only as the mass flow rate $V = q/L\rho_V$ [$\mathrm{(kcal/(m^2 \cdot h))(kg/kcal)(m^3/kg) = m/h}$], usually called the vapor-formation rate, and there is no cross section f of the moving flow. Therefore, for the process in tubes, Re* accounts only for the effect of the vapor-formation rate and cannot account for the effect of velocity of motion of the vapor–liquid flow. Under operating conditions, with constant specific heat flux along a vertical tube with lateral circulation of a boiling flow, the vapor-formation rate and Re* will be constant at each tube section, while the rate of vapor motion will increase along the tube because of the increase of vapor-content at successive sections, and the absolute velocity of motion of the two-phase boiling flow will in-

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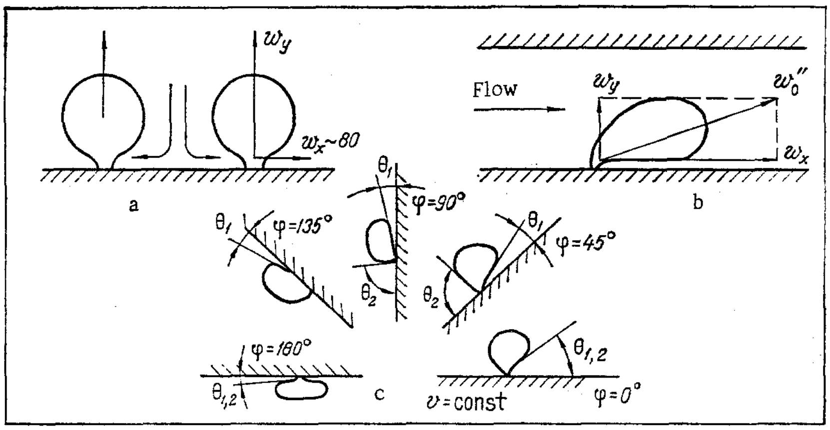

Fig. 1. Physical model of the motion of vapor bubbles during boiling: a) in a large volume [$w_y = w'' \simeq w_0'' = (q \cdot 80)/L\rho_v$; $w_x \sim 80$]; b) in tubes and channels [$w_y = q/L\rho_v$; $w_0'' = q(l/d)L\rho_v$; $w_x \sim l/d$]; c) on inclined surfaces.

crease continuously from section to section. Thus, the Reynolds analogy [Eq. (2)], which is suitable for bubble boiling of a liquid in a large volume on a horizontal surface, is not suitable for the hydrodynamics of a flow with boiling in tubes and channels. Therefore, the question of an analogy for boiling in tubes and channels requires special consideration.

We may note the following characteristic features of a physical model of the process of bubble boiling in tubes and in a large volume. In bubble boiling in a large volume on a horizontal surface, a bubble of vapor formed above the surface (Fig. 1a) will migrate from its formation center in the direction normal to the heated surface. The heat- and mass-transfer mechanisms will act preferentially in the same direction, along with all the forces perturbing the wall layer, into which the basic thermal resistance is concentrated. Here we can neglect mutual influence of neighboring bubbles, assuming these to be spherical with symmetrical contact angles, and we can consider the process to be self-similar with respect to the geometric size of the heated surface.

With boiling in tubes (Fig. 1b) and on inclined surfaces in a large volume (Fig. 1c) the vapor bubbles depart from their formation centers tangentially to the heated surface. The heat- and mass-transfer mechanisms and the perturbations in the wall layer depend mainly on the tangential stresses. In this case the tangential forces produce deformation of the bubble and asymmetry of the contact angles so that the mutual influence of neighboring bubbles can lead to their earlier fusion and to a breakdown of the hydrodynamic stability of the process. Under these conditions the boiling process is not self-similar in regard to the geometric dimensions and toward different orientations of the heat-transfer surface.

The directed motion of two-phase flow along the heat-transfer surface with boiling in tubes leads to a continuous increase in vapor content and to an increase in the speed of motion of the two-phase flow along the tube. With boiling in tubes the geometric dimensions affect the hydrodynamics and the heat transfer.

According to our experiments on boiling in tubes with natural circulation and constant q along the tube, an increase in geometrical dimension l/d leads to an increase in the tangential perturbations of the wall layer, to an increase in the flow speed by a factor of more than 100 (from 0.25 m/sec at the entrance to 30 m/sec at the exit), and to an increase in the average heat transfer by a factor of more than 2 [2-4].

The influence of these characteristics of the physical model and of the geometric dimensions on the boiling process has been confirmed also by the investigations of Fush, Tananaiko, and Grigor'ev et al.

The most popular (of the existing) correlations for heat transfer with boiling [5, 6, 8] has, unfortunately, several defects. One type of correlation, based on thermodynamic similarity theory, does not account for the hydrodynamics of the boiling process, and is therefore used only for boiling in a large volume. Another type of correlation, based on processing by methods of the theory of similarity between the equations of motion of mass and heat flux, does not consider substances in thermodynamically similar conditions and only accounts slightly for the hydrodynamic stagnation during boiling in tubes.

In order to derive a general parametric relation, based on our physical model of the process, we examine the following initial system of equations, used in [5-9].

1. The thermal-conduction equation for the liquid phase

$$\operatorname{div}(\lambda \operatorname{grad} T) = c\rho \frac{DT}{d\tau}. \tag{3a}$$

2. The equation of motion and continuity in the liquid phase

$$\begin{gathered} g\rho + 2\operatorname{div}(\mu\dot{S}) - \operatorname{grad}\left(P + \frac{2}{3}\mu \operatorname{div}\mathbf{w}\right) = \rho \frac{D\mathbf{w}}{d\tau}; \\ \operatorname{div}(\rho\mathbf{w}) + \frac{\partial\rho}{\partial\tau} = 0. \end{gathered} \tag{3b}$$

Here $\dot{S}$ is the strain rate tensor.

3. The equation of motion and continuity of the vapor phase, taking into account that grad $P'' =$ grad $P''_d + g\rho$ and $\rho_V =$ const.

$$\begin{gathered} g(\rho_V - \rho) - \operatorname{grad} P''_d + 2\operatorname{div}(\mu S) - \operatorname{grad} \frac{2}{3}\mu'' \operatorname{div}\mathbf{w}'' = \rho_V \frac{D\mathbf{w}''}{d\tau}; \\ \operatorname{div}(\rho_V \mathbf{w}'') + \frac{\partial\rho_V}{\partial\tau} = 0. \end{gathered} \tag{3c}$$

Here P''_d is the dynamic pressure component; $g\rho$ is the static pressure component.

4. The equation of thermal interaction at the phase boundary

$$\begin{gathered} -\lambda\left(\frac{\partial T}{\partial n}\right)_b = L\rho'' w''_V; \\ T''_b = T'' + \frac{AT''(\rho - \rho_V)}{L\rho\rho_V}\left[\frac{2\sigma\rho}{R(\rho - \rho_V)} - \frac{\lambda}{L}\left(\frac{\partial T}{\partial n}\right)_b \sqrt{2\frac{\pi P}{\rho_V}}\right]. \end{gathered} \tag{3d}$$

5. The conditions for mechanical interaction at the phase boundary

$$\begin{gathered} P''_b = P_b + \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)_{n_i} + (P^{(1)}_{i\kappa} - P^{(2)}_{i\kappa})_{n\kappa} + \frac{\partial\sigma}{\partial x_i}; \\ \mu\left(\frac{\partial w}{\partial y}\right) - \rho(\overline{v_i v_\kappa})_b = \mu''\left(\frac{\partial w''}{\partial y}\right) - \rho_V(\overline{v_i v_\kappa})_b; \\ (w''_{xz})_b = (w_{xz})_b \simeq 0. \end{gathered} \tag{3e}$$

6. The scale of vapor bubbles formed

$$\begin{gathered} R_0\sqrt{g\frac{\rho - \rho_V}{\sigma}} = f\left(\Theta; \frac{w^2}{gl_0}; \ldots\right); \\ \frac{\delta_i}{l_0} \ldots; \; n_i \ldots \end{gathered} \tag{3f}$$

7. The equation of heat transmission, allowing for the law of appropriate states

$$\alpha = f\left(\frac{RgP_{cr}}{MT_{cr}}\right) \cdot 2f_1(q) \cdot f_2\left(\frac{T}{T_{cr}}; \frac{P}{P_{cr}}; \frac{C_v}{R}; \ldots\right). \tag{3g}$$

The equations of motion and continuity [(3b) and (3c)], strictly speaking, have been written for motion of a single-phase nonheated flow, when the most important three-dimensional boundary condition is the flow velocity directly at the boundary surface, while the normal component of the velocity at the wall is zero.

But for the case of boiling, where there is absorption of the liquid phase and generation of the vapor phase, due to heating and physical change of phase at the heated surface, the motion of the flow at the hot boundary surface is dependent both on the normal and the tangential components of the vectorial velocity of motion. The normal component here is defined as the speed of the vapor-generation process at the wall $q/L\rho_V$. The tangential component is defined to be the rate of flow of liquid in the wall layer toward the vapor-generation site.

For boiling in a vertical tube with natural circulation the tangential component of the vectorial velocity of motion in the wall layer is determined by the combined effect of the hydrostatic pressure (the apparent level) and the velocity of motion of the vapor along the tube due to increase of vapor content. It is easy to see that the change in vapor content and the speed of motion of the vapor along the tube, for a given constant rate of vapor generation $q/L\rho_V$, is determined by the geometric parameters l/d or F/f [2, 3, 4].

For boiling in a large volume on a horizontal surface the tangential component of the velocity of motion also acts in the wall layer at each vapor-generation center (Fig. 1a). However, this quantity is appreciably less than the normal component and is determined by the effect of inflow into the circulation of mass, depending on the viscous properties of the liquid and the vapor.

In order to account for these effects in motion of boiling flows, we introduce additional boundary conditions into the system of equations (3) in the form of components of the velocity gradient in the boundary layer:

$$\left(\frac{\partial w_y}{\partial y}\right)_{\text{st}}=f\left(\frac{q}{L\rho_V}\right);\quad \left(\frac{\partial w_x}{\partial x}\right)_{\text{st}}=f_1\left(\frac{h}{l};\ l/d\right);\quad \left(\frac{\partial w_z}{\partial z}\right)_{\text{st}}=0 \tag{3h}$$

or in the form of components of the vectorial velocity of motion

$$w_y\sim\pm\frac{q}{L\rho_V},\quad w_x\sim\left(\frac{h}{l};\ \frac{l}{d}\right),\quad w_z=0.$$

With the original system of basic equations (3a)-(3h) one can quite accurately describe the hydrodynamic, thermal, and thermodynamic conditions for the boiling process in various situations. By treating this system of equations by the methods of similarity theory [5-7, 11, 12], we obtain the following correlation for a given heat flux density:

$$\begin{gathered}\frac{\alpha}{\lambda}\left[\frac{\sigma}{g(\rho-\rho_V)}\right]^{0.5}=f\left\{\frac{\nu}{a};\ \frac{g}{\nu^2}\left[\frac{\sigma}{g(\rho-\rho_V)}\right]^{1.5}\left(1-\frac{\rho_V}{\rho}\right);\right.\\ \frac{w_0 l_0}{\nu};\ \frac{w_0'' l_0}{\nu};\\ \frac{(L\rho_V)^2}{cT_V\rho\,[g\sigma(\rho-\rho_V)]^{0.5}};\ \frac{P}{[g\sigma(\rho-\rho_V)]^{0.5}};\quad M;\ \frac{T}{T_{\text{cr}}};\ \frac{P}{P_{\text{cr}}};\ \frac{h}{l};\\ \left.\frac{\sigma}{g(\rho-\rho_V)\,l_0^2};\quad \Theta;\quad \delta_i\left[\frac{g(\rho-\rho_V)}{\sigma}\right]^{0.5};\quad n_i\right\}.\end{gathered} \tag{4}$$

Analysis of Eq. (4) shows that the last four parameters may be neglected, because their effect is small, because there are no experimental data, or because one cannot evaluate the roughness of the heated surface (especially inside the tube).

In experimental practice it is very difficult to determine the effect of the parameters $\Pr=\nu/a$ and $\text{Ar}=(g/\nu^2)[\sigma/g(\rho-\rho_V)]^{1.5}[1-(\rho_V/\rho)]$, $\text{Kp}=P/[g\sigma(\rho-\rho_V)]^{0.5}$, and $K_i=(L\rho_V)^2/cT_V\rho[g\sigma(\rho-\rho_V)]^{0.5}$, and also T/T_{cr} and P/P_{cr} on the value of Nu, since there is almost a single-valued relationship between these parameters for most boiling substances. Ordinarily use is made of one of them.

The parameter

$$T_\tau=T/T_{\text{cr}} \tag{5}$$

(the relative or reduced temperature) is a thermodynamic similarity parameter for boiling material. It describes the relative kinetic energy of thermal motion of molecules of a substance at the given boiling temperature as a ratio to the energy or critical (thermodynamic) temperature. Thus, it expresses the thermal properties of the substances being prepared in the corresponding thermodynamic states. The parameter T_τ shows, e.g., that in the thermodynamic sense, helium, boiling at $P = 1$ bar and $T = 4.2°K$, can be compared with water boiling at $P = 41$ bar and $T = 524°K$, but not at $P = 1$ bar and $T = 373°K$. The parameter T_τ serves as a scale for determining heat transfer as we convert from one boiling substance to another.

The factor T is similar to K_t, K_p, and P/P_{cr} in regard to its effect on heat transfer during boiling. However, compared with the others, it has a certain advantage. It is simpler than the parameters K_t and K_p, and compared with P/P_{cr} it allows one to describe the heat-transfer process over the entire range of change of the operating parameters from $P = 1$ to P_{cr} by a simple power law, which cannot be done using parameters containing the pressure.

It has been determined from analysis of the experimental data of the various investigators that $\bar{\alpha} \sim T^7_\tau$ for heat transfer during boiling of cryogenic liquids: helium, hydrogen, neon, argon, oxygen, methane, and nitrogen (Fig. 2); $\bar{\alpha} \sim T^{10}_\tau$ for boiling of high-temperature liquids; water, alcohol, benzene, carbon dioxide, Freon-12, Freon-14, and others; and that $\alpha \sim T^8_\tau$ for boiling of metals: calcium, sodium, cesium, and mercury.

The high exponent of T_τ is probably linked to the intermolecular interaction energy, which depends on variation of the distance between the molecules of the material at different temperatures. It is known that the molecular interaction potential depends on the distance between the molecules to the powers 12 and 6, e.g., in the Lennard-Jones formula

$$E_{(r)} = 4\varepsilon\left[\left(\frac{u}{r}\right)^{12} - \left(\frac{u}{r}\right)^{6}\right].$$

It is known that in dealing with the equations of motion of the type (3b) or (3c) by the methods of similarity theory, following conversion to the new variables and multiplying each term by $w^2/l_0\rho$, we obtain the Reynolds number in general form

$$\mathrm{Re} = \frac{wl_0\rho}{\mu} = \frac{wl_0}{\nu}.$$

Here w is the characteristic flow velocity; l_0 is a characteristic dimension.

To describe the bubble boiling process the separated diameter of a vapor bubble

$$l_0 = d_0 = \left[\frac{\sigma}{g(\rho - \rho_V)}\right]^{0.5}$$

is usually taken as the characteristic dimension.

From an analysis of experimental investigations and from the physical model of the bubble boiling process in tubes with natural flow circulation we assumed the reduced velocity of vapor motion in the tube $w = w_0'' = (q/L\rho_V)\cdot(l/d)$ [2, 13] as the characteristic velocity. An experimental verification of this choice is the data of Table 1, from which it can be seen that at a constant rate of vapor generation the average heat-transfer coefficient for boiling in tubes increases by more than a factor of 2 with increase of the geometric dimensions l/d. The liquid velocity at the entrance and the reduced liquid velocity at the exit vary insignificantly. However, the reduced velocity of vapor motion at the tube exit increases by a factor of 6. These data show that the characteristic velocity for describing the hydrodynamics and the heat transfer during boiling in tubes with natural circulation is the reduced vapor velocity and not the rate of vapor generation.

The effect of geometric dimensions of the tubes on the heat transfer in boiling (Fig. 3) has been investigated experimentally [2-4, 16]. With increase of l/d up to 80, the heat transfer does not change and it is equal to the heat transfer for boiling in a large volume. With further increase of l/d from 80 to 560, the heat-transfer coefficient increases proportionally $\bar{\alpha}_{\mathrm{boil}} \sim (l/d)^{0.65}$.

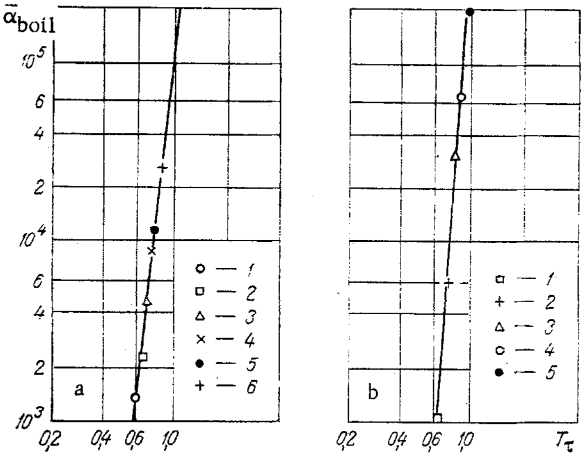

Fig. 2. Heat transfer during boiling as a function of $T_\tau = T/T_{cr}$: a) boiling of nitrogen; 1) P = 0.42 bar, Lyon; 2) 1 bar, Vishnev and Ivanov; 3,4,5,6) P = 2, 3.8, 7.7, 15.7 bar, Lyon; the curve shows $\bar{\alpha} \sim T^7_\tau$ for $w_0'' = 0.5$ m/sec = const; b) boiling of water, 1,2,3,4,5) P = 1, 10, 50, 100, 199 bar, Borishanskii; the curve shows $\bar{\alpha} \sim T^{10}_\tau$ for $w_0'' = 0.2$ m/sec = const, $\bar{\alpha}_{boil}$ W/(m²·°K).

TABLE 1. Heat Transfer for Boiling of Oxygen in Tubes at P = 1 bar as a Function of Various Characteristic Velocities

Tube dimension l/d	Heat flux q, W/m²	Heat-transfer coeff. $\bar{\alpha}$, W/(m²·°K)	Vapor generation rate $w'' = q/L\rho_v$, m/sec	Liquid velocity at entrance w_0, m/sec	Reduced velocity	
					of the liquid at entrance w_0', m/sec	of the vapor at exit $w_0'' = (ql/d)/L\rho_v$, m/sec
106	1290	722	0,0017	0,212	0,208	0,667
189	"	1230	"	0,378	0,368	1,23
560	"	1500	"	0,392	0,360	3,98
106	1720	950	0,0022	0,220	0,206	0,89
189	"	1430	"	0,382	0,372	1,64
560	"	1800	"	0,380	0,346	5,15

The physical cause for the effect of geometrical tube dimension on the heat transfer in boiling is the increase in reduced vapor velocity at each tube section located above the point. This velocity affects a change in the two-phase flow structure and increases the turbulence of the wall layer of liquid due to an increase in the tangential component.

From what has been said it becomes clear that, in the Reynolds analogy, one should take the reduced vapor velocity

$$w_0'' = \frac{q(l/d)^{0.65}}{L\rho_v}.$$

as the characteristic velocity for the boiling process in tubes with natural flow circulation.

The changed analog of the Reynolds number

$$W = \frac{w_0'' l_0}{\nu} = \frac{\dfrac{q(l/d)^{0.65}}{L\rho_v}\left[\dfrac{\sigma}{g(\rho-\rho_v)}\right]^{0.5}}{\nu} \qquad (6)$$

for the hydrodynamic characteristic of boiling flow in tubes has the same velocity structure

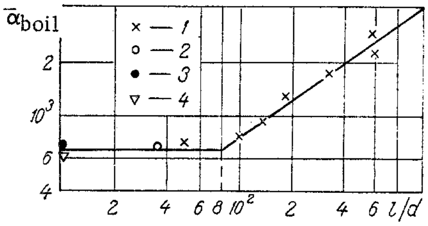

Fig. 3

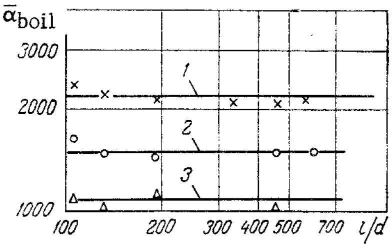

Fig. 4

Fig. 3. The heat transfer in boiling of oxygen as a function of the geometrical dimensions of the tubes, with constant rate of vapor generation and $q = 1000$ W/m² = const, P = 1 bar: 1) in tubes with l/d = 560, 330, 189, 132, 106, 51, Vishnev; 2,3,4) in a tube bundle with pitch t = 11 mm (l/d = 35), t = 20 mm (l/d = 6.3) and in a large volume, Ivanov. α_{boil}, W/(m²•°K).

Fig. 4. Heat transfer in boiling of oxygen as a function of geometrical dimensions of the tubes, with constant vapor velocity w_0'': 1) w_0'' = 4.4 m/sec; 2) 2; 3) 1.

$$w = \frac{V}{f} = \frac{\frac{q}{L\rho_V} F}{f} = \frac{q\frac{F}{f}}{L\rho_V} = \frac{q\,4l/d}{L\rho_V}$$

as does the Reynolds number (1). It is more complete than the Re* analog and covers typical aspects of the hydrodynamic process by allowing for the effect of the velocity components.

It can be seen that physically Eq. (6) is a measure of the ratio of inertial forces to molecular friction forces and, in addition, it accounts for the effect of the vectorial components of the inertial forces. The group

$$\frac{ql/d}{L\rho_V} = f\left(\frac{w_y}{w_x}\right) = \frac{q/L\rho_V}{d/l} = \frac{q/L\rho_V}{f/F},$$

which appears in the criterion, defines the ratio of the normal component of the vapor velocity vector $q/L\rho_V$ to the quantity d/l, which describes the tangential component of the vapor velocity vector in the tube. The quantity l/d reflects the influence of vapor evacuation conditions on the vapor-generating center, variation in the vapor content, and variation in the vapor velocity along the tube [4].

In an attempt to show the nature of this parameter, we denote it by the symbol W.

The curve in Fig. 3 shows that in tubes with $l/d > 80$ the tangential component of the vapor velocity vector has a dominant effect on the flow hydrodynamics at the heated surface and, therefore, on the molar and molecular heat transfer in boiling. In tubes with $l/d < 80$ the normal velocity component has a governing effect on the process, and heat transfer in boiling in tubes with $l/d < 80$ becomes the same as in a large volume. Therefore, for boiling on a horizontal surface in a large volume one should use a constant coefficient, equal to 80, in Eq. (6), instead of l/d. Physically, this factor describes the constant displacement of liquid at the vapor-generating center due to the tangential component. This displacement depends on the viscous properties of the boiling medium. It is known that the maximum rate of natural motion of vapor bubbles in water, associated with the viscous properties of the vapor—liquid system, is 0.4–0.45 m/sec. The knee point at $l/d = 80$ in the curve in Fig. 3 also corresponds to the velocity $w_0'' \simeq 0.45$ m/sec.

Thus, the parameter W can be said to characterize the hydrodynamics of the boiling process, not only in tubes, but also in a large volume. The parameter Re* is a special case of W, since it accounts for the effect of only one component (the normal) of the vapor velocity vector.

The parameter W, being an independent parameter, cannot be represented as the product $W = \text{Re}^* \cdot l/d$, since the first factor Re*, as was shown above, cannot describe the process of boiling in tubes. The second factor l/d, as can be seen from Fig. 4, also cannot independently describe the effects of hydrodynamics on boiling heat transfer. For constant reduced vapor velocity $w_0'' = \text{const}$, the heat transfer for boiling in tubes is also constant, in spite of the variation of l/d from 100 to 560. Experimental results show that the effect of l/d only appears in terms of the reduced velocity of vapor motion (Fig. 4).

Thus, a formal separation of the parameter l/d from the parameter W is contradicted by the experimental data and constitutes a denial of the characteristic reduced vapor velocity as determining the process of tube boiling (Table 1). In this respect W conforms to the Archimedes parameter, which also cannot be represented as the product $\mathrm{Ar} = \mathrm{Ga}(\Delta\rho/\Delta)$.

The governing parameter

$$\mathrm{H} = \frac{h}{l} \tag{7}$$

is the relative (apparent) liquid level in the evaporator tube. It describes the effect of the hydrostatic level on the hydrodynamics of two-phase flow with boiling under natural circulation conditions, when the liquid velocity at the tube entrance w_0 is not given. Physically, it describes the effect of the ratio of the heating zone and the zone of developed boiling on the average heat-transfer coefficient in the tube, accounting for the heat flux and the geometric dimensions of the tube $\bar{\alpha} \sim \mathrm{H}^{-m}$.

It has been established, from experimental data [2-4, 13] on boiling of oxygen, nitrogen, and a mixture of the two in tubes, that the exponent of the hydrostatic parameter for cryogenic liquids has the form

$$m = [2900\,(l/d)^{-1.65}]^{1-k}\cdot 10^{-k}\,, \tag{8}$$

where $k = q/0.1\,q_{\mathrm{cr}_1}$.

In designing equipment it is appropriate to determine the exponent from the graph in [13].

For a heat flux density $q \geq 0.1\,q_{\mathrm{cr}}$, and also for $l/d \geq 500$, the effect of H on heat transfer in boiling degenerates. For liquid boiling in a large volume and in tubes with relative level $h/l \geq 0.75$, the parameter $\mathrm{H} \simeq 1$.

From the experimental data of Tolubinskii et al. [14, 15] on boiling of water, we find that the exponent m for high-temperature liquids is given by the formula.

$$m_1 = [1300\,(l/d)^{-1.65}]^{1-k}\cdot 10^{-1.35k}. \tag{9}$$

The parameters W and H can be used to combine hydrodynamics and heat transfer for boiling in tubes and channels with boiling in tube bundles and in large volumes into a single law, and also to design types of equipment which exhibit these specific features, using a single technique and a single general formula.

The analysis of the systems of equations (3a)-(3i) and Eq. (4) yields the following parametric equation for bubble boiling of liquids in a large volume and in tubes with natural flow circulation:

$$\mathrm{Nu} = f\,(\mathrm{M};\ \mathrm{Pr};\ \mathrm{W};\ \mathrm{H};\ \mathrm{T}_\tau). \tag{10}$$

NOTATION

l, d, f, length, internal diameter, and cross section of the tube; q, q_{cr_1}, heat flux and first critical heat flux for boiling; ρ, ρ_v, ρ'', densities of the liquid and the vapor; L, heat of vapor formation; g, acceleration of gravity; T, T_{cr}, boiling temperature and critical temperature; σ, coefficient of surface tension; $w_0'' = q[(l/d)/L\rho_v]$, reduced vapor velocity; λ, ν, thermal conductivity and kinematic viscosity of the liquid; τ, time; M, molecular weight of the substance; Pr, Prandtl number; Nu, Nusselt number.

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THEORY OF DEEP DRYING OF FINELY POROUS BODIES AND EXPERIMENTAL VERIFICATION OF THEORY

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Approximate formulas for calculating the initial phase of the deep drying of finely porous bodies are obtained for the case of random adsorption isotherms. Examples of calculations using these formulas showing a satisfactory agreement with experimental data are given.

Formulas for the coefficients of internal mass transfer accompanying the deep drying of finely porous bodies under isothermal conditions are derived in [1] on the basis of the kinetics of desorption processes. The next problem is that of calculating the rates and duration of isothermal deep drying up to a given final mass content u_F. In order to simplify the calculations it is convenient to approximate the actual continuous adsorption isotherms by broken isotherms comprising n links [2-4]. The initial section of the isotherm, corresponding to the deep-drying region, can be replaced with adequate accuracy [5] by a broken line in two links (n = 2). The equation for the isotherm is written in this case as follows:

$$a = a_1 = \beta_1 c, \quad 0 \leqslant c \leqslant c_*; \quad a = a_2 = \beta_1 c_* + \beta_2 (c - c_*), \quad c \geqslant c_*. \tag{1}$$

Here β_1 and β_2 are the isotherm parameters: when $\beta_1 > \beta_2$, the isotherm is convex, and when $\beta_1 < \beta_2$, it is concave.

In [2, 4], in particular, the problem of the kinetics of desorption from a porous half-space is examined for an isotherm of type (1). When the coefficient of internal diffusion D_i = const, it is formulated as follows:

$$\begin{aligned} (1+\beta_1)\frac{\partial c_1}{\partial \tau} &= D_i \frac{\partial^2 c_1}{\partial x^2}, \quad 0 \leqslant x \leqslant l(\tau); \\ (1+\beta_2)\frac{\partial c_2}{\partial \tau} &= D_i \frac{\partial^2 c_2}{\partial x^2}, \quad l(\tau) \leqslant x < \infty; \end{aligned} \tag{2}$$

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